

Combined Gas Law, Ideal Gas Law, Graham's Law & Root Mean Square Velocity**Combined Gas Law**

The first law for today is the **Combined Gas Law**. As the name implies, this law combines a number of the laws we studied earlier, specifically Boyle's Law, Charles' Law and Gay-Lussac's Law. The formula for the Combined Gas Law is:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Again, it is of the utmost importance that matching variables have the same units; meaning, if the initial pressure is in atm and the final pressure is torr, you have to convert one of the variables so that it matches the other. The same is true about volume. Temperature must always be in Kelvin.

Example: The volume of a gas-filled balloon is 30.0 L at 40 °C at 150. kPa pressure. What volume will the balloon have at standard temperature and pressure (STP)?

Standard temperature is 0 °C; standard pressure is 101.3 kPa.

P₁: 150. kPa

P₂: 101.3 kPa

V₁: 30.0 L

V₂: X

T₁: 40 °C + 273 = 313 K

T₂: 0 °C + 273 = 273 K

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{(150.)(30.0)}{313} = \frac{(101.3)(X)}{273}$$

$$\begin{aligned} (150.)(30.0)(273) &= (101.3)(X)(313) \\ 1228500 &= 31706.9 X \\ \mathbf{38.7 L} &= \mathbf{X} \end{aligned}$$

Ideal Gas Law

The formula we will use for the ideal gas law is: **PV = nRT**

P: pressure, can be in kPa or atm

V: volume, must be in liters (L) or cubic decimeters (dm³) – both units mean the same thing

n: moles

R: ideal gas constant (named in honor of Regnault, a French physicist) = 8.3145 (L kPa)/(mol K) or 0.08206 (L atm)/(mol K)

T: temperature, must be in Kelvin

Please note that if your pressure is in kPa, you **MUST** use 8.3145(L kPa) / (mol K) as your value for R. If your pressure is in atm you **MUST** use 0.08206 (L atm) / (mol K) as your value for R.

Let's try a sample problem. You fill a rigid steel cylinder with a volume of 20.0 L with nitrogen gas to a final pressure of 20,000. kPa at 27 °C. How many moles of N₂ gas does the cylinder contain? How many grams of N₂ are in the container?

P: 20,000. kPa

PV = nRT

V: 20.0 L

$$(20,000.)(20.0) = (X)(8.3145)(300)$$

n: X

$$400,000 = 2493 X$$

R: 8.3145 L kPa/mol K

$$\mathbf{161 \text{ moles} = X}$$

T: 27 °C + 273 = 300. K

To change our answer from moles to grams we have to multiply by the molar mass. (When going from grams to moles you would divide by the molar mass.)

$$N_2: 2 \times 14.0 = 28.0 \text{ g/mol}$$

$$161 \times 28.0 = \mathbf{4510 \text{ grams}}$$



Calculating Gas Density

Density is defined as the ratio of the mass of a substance to its volume. An important use of the ideal gas law is to determine the molar mass of a gas from its measured density. By manipulating the ideal gas law formula you can come up with: $(MM)P = dRT$; MM = molar mass, P = pressure, d = density, R = ideal gas law constant and T = temperature in Kelvin

Example: A gas has a density of 1.87 g/L at a pressure of 1.60 atm and 23°C. Calculate the molar mass of the gas.

$$\text{Molar Mass} = dRT$$

$$(MM)1.60 = (1.87)(0.08206)(296)$$

$$MM = 28.4 \text{ g/mol}$$

Graham's Law of Effusion

Graham's Law is named for Scottish chemist Thomas Graham (1805 – 1869). Graham studied rates of **effusion**, which occurs as a gas escapes through a tiny hole in a container of gas. Graham observed that the lower the molecular mass, the faster the gas effused. Further investigation led to **Graham's Law of Effusion**: the rate of effusion is inversely proportional to the square root of its molar mass. Later, it was found that this law was also applicable to **diffusion**, the tendency of molecules and ions to move from an area of high concentration to areas of low concentration until the concentration is uniform throughout the system. An example of **diffusion** would be breaking a bottle of cologne and having the smell spread throughout the room. An example of **effusion** would be a gas pouring out of a hole in a balloon or tire.



The formula for Graham's Law is:

$$\text{Rate}_A / \text{Rate}_B = \sqrt{\text{molar mass}_B} / \sqrt{\text{molar mass}_A}$$

Example: Compare the rates of effusion of nitrogen and helium. If helium takes 20 seconds to effuse, how long will it take for nitrogen to effuse?

$$\text{Rate}_A / \text{Rate}_B = \sqrt{\text{molar mass}_B} / \sqrt{\text{molar mass}_A}$$

Substance A: He: molar mass = 4.0 grams

Substance B: N₂: molar mass = 28.0 grams

(Hint: Always call your lighter molecule Substance A)

$$\text{Rate}_A / \text{Rate}_B = \sqrt{28.0} / \sqrt{4.0}$$

$$\text{Rate}_A / \text{Rate}_B = 5.291575 / 2.0$$

$$\text{Rate}_A / \text{Rate}_B = 2.6$$

Helium effuses 2.6 times faster than nitrogen at the same temperature.

20 seconds x 2.6 = 52 seconds. N₂ takes 52 seconds to effuse.

Root Mean Square Velocity

Root mean square velocity, $v_{rms} = \sqrt{3RT/M}$ R = 8.3145 J/mol K, T = temperature, M = mass of a mole in kilograms, also important: J = Kg m²/s². The average distance a particle travels between collisions in a particular gas sample is called the mean free path. It is typically a very small distance (1 x 10⁻⁷ m for O₂ at STP).

Example: Calculate the root mean square velocity for the atoms in a sample of helium at 25°C.

$$M = \frac{4.00 \text{ grams}}{\text{mole}} \times \frac{1 \text{ kilogram}}{1000 \text{ grams}} = 4.00 \times 10^{-3} \text{ kg/mole}$$

$$T = 25 + 273 = 298 \text{ K}$$

$$v_{rms} = \sqrt{3RT/M}$$

$$v_{rms} = \sqrt{3(8.3145 \text{ J/mol K})(298 \text{ K})/4.00 \times 10^{-3} \text{ kg/mol}}$$

$$v_{rms} = \sqrt{1.86 \times 10^6 \text{ m}^2/\text{s}^2}$$

$$v_{rms} = 1.36 \times 10^3 \text{ m/s}$$

Homework:

Combined Gas Law: Fill in the missing information.

	P₁	V₁	T₁	P₂	V₂	T₂
1	600. mm Hg	2.5 L	22 °C	760 mm Hg	1.8 L	270 K
2	1.2 atm	750 mL	0 °C	2.0 atm	500. mL	25 °C
3	95 kPa	4.0 L	3.0 x 10² K	101 kPa	6.0 L	471 K

4. The volume of a gas-filled balloon is 50.0 L at 20. °C and 742 torr. What volume will it occupy at standard temperature and pressure (STP)?

46 L

5. 15.00 liters of gas at 45.0 °C and 800. torr is heated to 400. °C and the pressure changed to 300 torr. What is the new volume?

84.7 L

Ideal Gas Law

1. What are the two values for R?

R= 0.08206 L atm / mol K or 8.3145 L kPa / mol K

2. How many moles of oxygen gas will occupy a volume of 2.5 liters at 1.2 atm and 25 °C?

0.12 moles

3. What pressure will be exerted by 25 grams of carbon dioxide at a temperature of 25 °C and a volume of 500. mL?

28 atm or 2800 kPa

4. At what temperature will 5.00 grams of Cl₂ exert a pressure of 900. torr at a volume of 750 mL?

154 K

5. What is the mass of 3.2 liters of oxygen at STP?

4.6 g

6. What volume will 454 grams of hydrogen gas occupy at 1.05 atm and 25°C?

5230 L

Gas Density

1. Calculate the density of nitrogen at 1.0 atm and 273 K.

1.3 g/L

2. Ammonia at has a density of 1.23 g/L at a temperature of -23°C. What is the pressure of the gas?

1.48 atm

3. Calculate the density of methane at STP.

0.717 g/L

4. At what temperature will carbon dioxide have a density of 2.49 g/L if the pressure is 1.23 atm?.

265 K

5. An unknown gas has a density of 0.16 g/L at a pressure of 0.98 atm and a temperature of 25°C. Determine the molar mass and identify the gas.

4.0 g/mol Helium

Graham's Law

1. What is effusion?

which occurs as a gas escapes through a tiny hole in a container of gas

2. What is diffusion?

the tendency of molecules and ions to move from an area of high concentration to areas of low concentration until the concentration is uniform throughout the system

3. Under the same conditions of temperature and pressure, how many times faster will hydrogen effuse compared to carbon dioxide?

$$\sqrt{44.01} / \sqrt{2.02}$$

4.7 times faster

4. If the carbon dioxide in problem 3 takes 32 seconds to effuse, how long will hydrogen take?

$$32 \div 4.7 = 6.8 \text{ sec}$$

5. What is the rate of diffusion of NH_3 compared to He? Does NH_3 effuse faster or slower than He?

$$\sqrt{17.04} / \sqrt{4.00} = 2.06 \text{ times slower}$$

6. If the He in problem 5 takes 20. seconds to effuse, how long will NH_3 take?

$$20 \times 2.06 = 42 \text{ sec}$$

7. An unknown gas diffuses 0.25 times as fast as He. What is the molecular mass of the unknown gas?

$$1.0 / 0.25 = \sqrt{x} / \sqrt{4.0}$$

64 grams

8. Hydrogen sulfide, H_2S , has a very strong rotten egg odor. H_2S particles travel at about 650. m/s. Methyl salicylate, $\text{C}_8\text{H}_8\text{O}_3$, has a wintergreen odor and benzaldehyde, $\text{C}_7\text{H}_6\text{O}$ has an almond odor. Calculate the rates of diffusion for both methyl salicylate and benzaldehyde.

$$650 / x = \sqrt{152.0} / \sqrt{34.1}$$

$\text{C}_8\text{H}_8\text{O}_3$ – 310. m/s

$$650 / x = \sqrt{106.0} / \sqrt{34.1}$$

$\text{C}_7\text{H}_6\text{O}$ – 370. m/s

Root Mean Square Velocity

1. Calculate the root mean square velocity of nitrogen at 25°C .

$$\sqrt{(3)(8.3145)(298)/0.02802}$$

515 m/s

2. At what temperature will oxygen have a root mean square velocity of 800. m/s?

$$800. = \sqrt{(3)(8.3145)(x)/0.03200}$$

$$640000 = (3)(8.3145)(x)/0.03200$$

$$20480 = (3)(8.3145)(x)$$

$$x = 821 \text{ K}$$

3. Calculate the root mean square velocity of sulfur dioxide at 225°C .

$$\sqrt{(3)(8.3145)(498)/0.06406}$$

440. m/s

4. At what temperature does helium have a root mean square velocity of 100.0 m/s?

$$100.0 = \sqrt{(3)(8.3145)(x)/0.00400}$$

$$10000 = 24.9435 x/0.00400$$

$$40 = 24.9435 x$$

$$x = 1.6 \text{ K}$$